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AKM equal to angle *EDB* or the points *B*, *D*, *M* and *K* are concyclic and therefore is angle *BMD*=angle *BKD*=90°.

This problem was also solved by *G. B. M ZERR*, *O. W. ANTHONY*, *E. W. MORRELL*, and *P. S. BERG*.

PROBLEMS.

48. Proposed by I. J. SCHWATT, Ph. D., Professor of Mathematics in the University of Pennsylvania, Philadelphia, Pennsylvania.

The Simson line belonging to one point of intersection of Brocord's Diameter, of a triangle with the circumcircle of this triangle is either parallel or perpendicular to the bisector of the angle formed by the side *BC* of the triangle *ABC* and the corresponding side B' C' of Brocard's triangle.

49. Proposed by J. C. WILLIAMS, Rome, New York.

Of all triangles inscribed in a given segment of a circle, with the chord as base, the isosceles is the maximum.

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CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

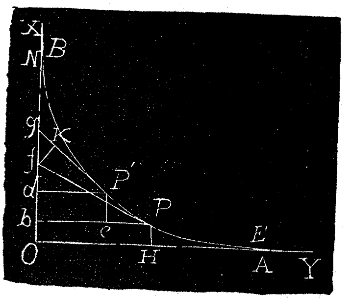
34. Proposed by GEORGE LILLEY, Ph. D., LL. D., Park School, Portland, Oregon.

A hare is at *O*, and a hound at *E*, 40 rods east of *O*. They start at the same instant each running with uniform velocity. The hare runs north. The hound runs directly toward the hare and overtakes it at *N*, 320 rods from *O*. How far did the hound run?

I. Solution by M. C. STEVENS, A. M., Professor of Mathematics, Purdue University, Lafayette, Indiana; COOPER D. SCHMITT, A. M., University of Tennessee, Knoxville, Tennessee; and G. B. M. ZERR, A. M., Ph. D., Inter States College, Texarkana, Texas.

Let u= velocity of hound, v= velocity of hare, $AO=a$, $BO=b$, $OH=x$, $PH=y$, $AP=s$, $\frac{v}{u}=n$. Then $nPA=Of$. $\therefore ns=y-x\frac{dy}{dx}$.

Differentiating with respect to x, we have $n\frac{ds}{dx}=-x\frac{d^2y}{dx^2}$. But s increases as x diminishes, whence $\frac{ds}{dx}=-\sqrt{1+\left(\frac{dy}{dx}\right)^2}$. $\therefore \frac{n}{x}$



$=\dfrac{\dfrac{d^2y}{dx^2}}{\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}}$. Integrating and noting that $x=a$, $\dfrac{dy}{dx}=0$, together, n

$\log\dfrac{x}{a}=\log\left[\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}+\dfrac{dy}{dx}\right]$. $\therefore \left(\dfrac{x}{a}\right)^n=\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}+\dfrac{dy}{dx}$, also by reciprocals, $\left(\dfrac{a}{x}\right)^n=\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}-\dfrac{dy}{dx}$. Addding we get $2\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}=2\dfrac{ds}{dx}$ $=\left(\dfrac{x}{a}\right)^n+\left(\dfrac{a}{x}\right)^n$. $\therefore\ s=\tfrac{1}{2}\displaystyle\int_0^a\left[\left(\dfrac{x}{a}\right)^n+\left(\dfrac{a}{x}\right)^n\right]dx=\tfrac{1}{2}\left[\dfrac{x^{n+1}}{a^n(n+1)}+\dfrac{a^nx^{1-n}}{1-n}\right]_0^a$ $=\dfrac{a}{1-n^2}$. $\therefore\ s=\dfrac{a}{1-n^2}$, but $ns=b$, $\therefore\ n=\dfrac{b}{s}$. $\therefore\ s=\dfrac{as^2}{s^2-b^2}$, $\therefore\ s^2-as=b$, $s=\tfrac{1}{2}[a\pm\surd(a^2+4b^2)]$ $s=\tfrac{1}{2}[a+\surd(a^2+4b^2)]$, the minus sign not being admissable. $s=\tfrac{1}{2}[40+\surd(1600+409600)]=340.624+$ rods. To find the rectangular equation to the curve, we have by subtracting $2\dfrac{dy}{dx}=\left(\dfrac{x}{a}\right)^n-\left(\dfrac{a}{x}\right)^n$. $\therefore\ 2(y+C)$ $=\dfrac{x^{n+1}}{a^n(n+1)}-\dfrac{a^nx^{1-n}}{1-n}$. When $y=0$, $x=a$, which gives $C=-\dfrac{an}{1-n^2}$.

$$\therefore\ 2\left(y-\frac{an}{1-n^2}\right)=\frac{x^{n+1}}{a^n(n+1)}-\frac{a^nx^{1-n}}{1-n}.$$

II. Solution by **ALFRED HUME, C. E., D. Sc., University of Mississippi, and the PROPOSER.**

The tangent to the path of the hound always passes through the position of the hare, the point of tangency being the simultaneous position of the hound. Let P and P' be two positions of the hound, PP' being infinitesimal, f and g corresponding positions of the hare. Draw the ordinates $P'd$ and Pb and the perpendiculars fk and $P'e$ to $P'g$ and Pb respectively. Let $AP=s$, $PP'=ds$, $OB=x$, $bd=eP'=dx$, $OF=z$, $fg=dz$. Now, since the two animals run with uniform velocities, $\dfrac{ds}{dz}=$ some constant$=l$. In the limit, △'s $P'eP$ and fgk are similar.

$$\therefore\ \frac{ds}{dx}=\frac{dz}{gk};\quad gk=dx\frac{dz}{ds}=\frac{dx}{l}.$$

The change in the length of the tangent as the hound runs from P to P' is due to a positive increment, gk, at one end, and a negative increment, PP', at the other. Therefore, if $t=$ the length of the tangent, $dt=\dfrac{dx}{l}-ds$, and, integrating, $t=\dfrac{x}{l}-s+C$. When $t=40$, $x=0$, and $s=0$; $\therefore\ C=40$. For

s substitute its value lz; then $t=\frac{x}{l}-lz+40$. When $t=0$, $x=320$, and $z=320$.

$$\therefore 0=\frac{320}{l}-320l+40, \text{ and } l=1.0644+. \quad s=lz=1.0644\times320=340.$$

624+, the distance in rods that the hound runs.

Also solved by *A. L. FOOTE*, *P. S. BERG* and *F. P. MATZ*.

35. **Proposed by H. C. WHITAKER, B. S., C. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.**

At the rate of 10 cubic inches per second, water is running into a vessel in the shape of a right conic frustum, the radii of whose upper and lower bases are respectively 15 and 10 inches, and whose altitude is 20 inches. At what rate per second is the *depth* of the water increasing, when *it* is exactly 8 inches?

I. **Solution by J. C. NAGLE, M. A., M. C. E., Professor of Civil Engineering, A. and M. College, College Station, Texas, and ALFRED HUME, C. E., D. Sc., University of Mississippi.**

Represent the depth and the volume of water at any instant by h and V respectively. The radius of the free surface of the water will be $10+\frac{h}{4}$.

Therefore $V=\frac{\pi}{3}h\left[100+\left(10+\frac{h}{4}\right)^2+10\left(10+\frac{h}{4}\right)\right]$.

Differentiating, $dV=\frac{\pi}{3}(300+15h+\frac{3}{16}h^2)$. Substituting for dV and h 10 and 8 respectively, $dh=\frac{5}{72\pi}$, or the depth is increasing at the rate of

$\frac{5}{72\pi}=0.02210484+$ of an inch per second.

[This result may also be obtained as follows: The area of the free surface of the water at the instant under consideration is 144π. Therefore the depth is increasing at the rate of $\frac{10}{144\pi}$ inches per second.]

II. **Solution by Professor J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.**

Let the radii of upper and lower bases be a and b, the altitude $=h$, $x=$ the distance of water from the lower base after the time t, y the radius of section at the distance x; then, denoting by c the quantity of water flowing in per second, $\pi y^2dx=cdt$.

$$\therefore \frac{dx}{dt}=\frac{c}{\pi y^2}=\frac{ch^2}{\pi[(a-b)x+bh]^2}.$$

For the given numerical values, we have $\frac{dx}{dt}=\frac{5}{72\pi}=$ required rate at distance $=8$.

Also solved by *P. S. BERG*, *F. P. MATZ* and *E. L. SHERWOOD*.